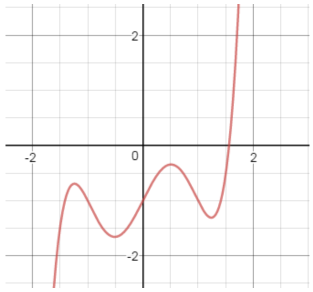
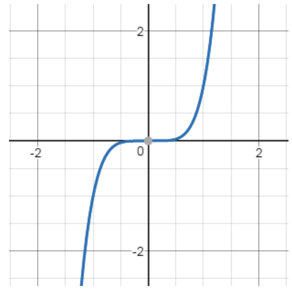
Functions - Graphs of Polynomials

END BEHAVIOR: Polynomials have the same end behavior of the basic function of the same degree.

Ie. $f(x) = 3x^5 - 4x^3 + 2x - 5$ has the same end behavior as $f(x) = x^5$.





EVEN and ODD polynomials:

Ex. Is $f(x) = 3x^5 - 4x^3 + 2x - 5$, even or odd?

$f(-x) = 3(-x)^5 - 4(-x)^3 + 2(-x) - 5 = -3x^5 + 4x^3 - 2x - 5 \neq -f(x)\ OR\ f(x)$, so NEITHER.

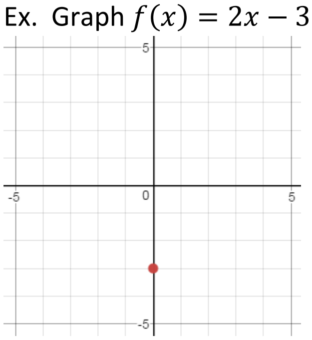
Ex. Is $f(x) = 3x^5 - 4x^3 + 2x$, even or odd?

$f(-x) = 3(-x)^5 - 4(-x)^3 + 2(-x) = -3x^5 + 4x^3 - 2x = -f(x)$, so ODD.

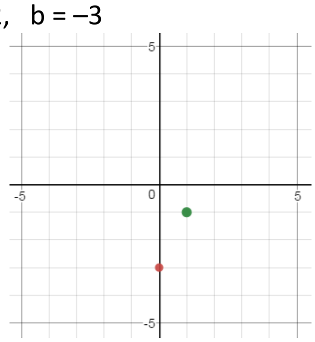
Note: A polynomial is odd if ALL of the terms are ODD powered.
A polynomial is even if ALL of the terms are EVEN powered.

Linear functions: $f(x) = mx + b.$ » $m = slope = \frac{rise}{run}$, $b = y - intercept$

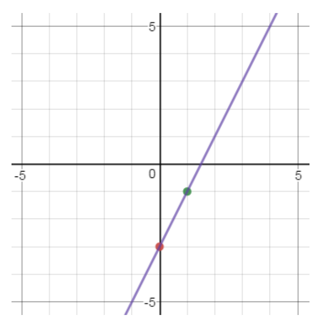
Ex. Graph $f(x) = 2x - 3$ m = 2, b = –3



Plot the y-intercept



Use the slope to find another point

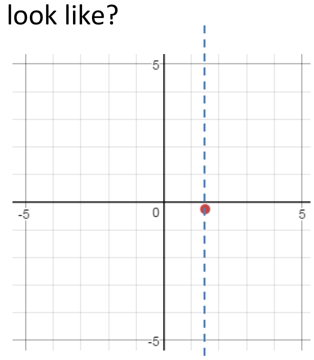


Connect the dots

The y-intercept is the value of y when x = 0.
Less used for lines is the x-intercept. The x- intercept is the value of x when y = 0.

Find the x-intercept of $f(x) = 2x - 3$.
Let $y = f(x) = 0$, so $0 = 2x - 3$. Solving for x, $3 = 2x$ $x = \frac{3}{2}$
So the x-intercept is $\frac{3}{2}$, which means the line crosses the x-axis at $\frac{3}{2}$, which it does on the graph.

Quadratic functions: $f(x) = x^2 - 3x + 2$ ❯ It is a parabola. But what does it look like?



STEP 1: For a quadratic equation of the form $f(x) = ax^2 + bx + c$,

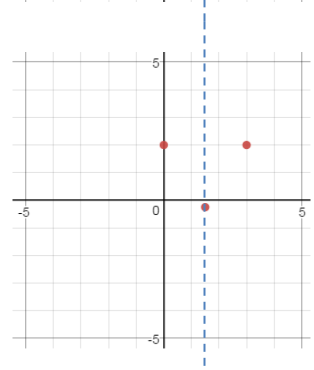
The maximum or minimum of the parabola occurs at $= \frac{-b}{2a}$.

For the example, a = 1 and b = –3, so the formula gives $x = \frac{-(-3)}{2(1)} = \frac{3}{2}$

Plugging in $\frac{3}{2}$ for x, $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2 = \frac{9}{4} - \frac{9}{2} + 2 = \frac{-1}{4}$

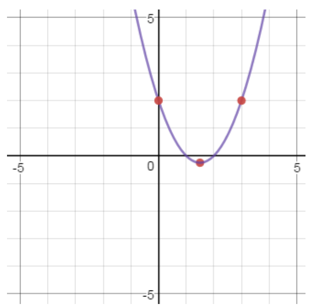
So the minimum occurs at $\left(\frac{3}{2}, \frac{-1}{4}\right)$

The parabola is symmetric about this point.

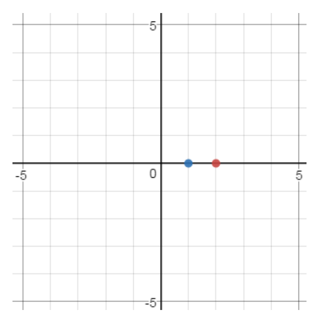


STEP 2: Find another point by picking a friendly x value and plugging it in.
Let $x = 0$, $f(0) = (0)^2 - 3(0) + 2 = 2$ so the parabola goes through (0, 2) and the point that is reflected across the line of symmetry.

STEP 3: Draw the parabola



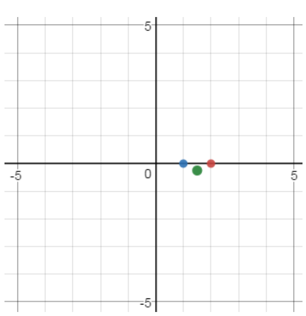
For quadratics and higher powered polynomials, it is helpful to find where the graph crosses the x-axis (the x-intercepts). These intercepts occur when the function equals zero and are called the ZEROS or ROOTS of the function.

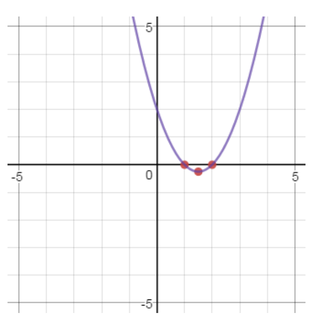


To find where $x^2 - 3x + 2 = 0$, factor the function: $x^2 - 3x + 2 = (x-2)(x-1) = 0$

From the Zero Factor Property, $x - 2 = 0, so\ x = 2$ & $x - 1 = 0, so\ x = 1$.

Then you can plot the minimum and then connect the dots





Higher Order polynomials:

Ex: $f(x) = 3x^5 - 4x^3 + 2x - 5$ How would you go about graphing this?

A note about the zeros of the function?

A polynomial has AT MOST as many zeros as its DEGREE.

So for a quadratic (degree 2), the parabola can cross the x-axis AT MOST twice.

For the example above, it is degree = 5, and could have 5 zeros. But it doesn't have to, so…bummer.

All we really know is the end behavior: $y \to -\infty \; as \; x \to \infty \; and \; y \to \infty \; as \; x \to \infty$

Other than that, the best you can do is make a table and hope for the best.